

ANALYSIS OF THE VISCOSITY OF SPECIFIC WAVES AND WAVE PROPAGATION
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Annotation. The main purpose of the article is a comparative study of the existence eigenwaves in an infinite viscoelastic waveguide of a sector cross section and a plate (based on the hypotheses of Kirchhoff and Timoshenko) of variable cross section (in the form of a wedge) depending on various parameters of the object (wave number and geometrical parameters). Using the Navier equations and the physical relation, a system of six differential equations in partial derivatives for a sector waveguide is obtained. After simple transformations, a system of differential equations with complex coefficients is obtained, which is further solved using the method of straight lines, which will allow using the software tool of the orthogonal sweep method with a combination of the Muller method on complex arithmetic in solving. Same, Based on the variational principle, a system of ordinary differential equations with complex coefficients is given. On the basis of numerical calculations, it has been established that the real parts of the phase velocity of propagation of the first mode are less from the Rayleigh wave velocity to 20%, on the segment $0^\circ < \varphi < 90^\circ$ (the central angle in the wedge-shaped waveguide), and further asymptotically approaches the Rayleigh wave, which propagates in the viscoelastic half-plane. Similar results were obtained for a wedge-shaped plate according to the theory of plates by Kirchhoff and Timoshenko, in the entire wave range. The results on the dynamic theory of elasticity and the plate (based on the hypotheses of Kirchhoff and Timoshenko) differ by no more than 6% for wedge apex angles not exceeding 28° . At $28^\circ < \varphi < 56^\circ$ the calculation results differ by up to 20%. It has been established that at small wedge angles, the simplified theory of Kirchhoff – Love and Timoshenko can be used in the entire wave range.

Keywords: fading wave, viscoelastic cylinder, sectoral cross section, Navier equation, spectral boundary value problem, orthogonal sweep.

**MUAYYAN TO‘LQINLARNING VISKOZITETI VA ULARNING TARQALISHINI
TAHLIL QILISH**

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Annotatsiya: Ushbu maqolaning asosiy maqsadi – cheksiz viskoelastik to‘lqin yo‘lovchi sektor kesimida va o‘zgaruvchan (ponasimon) kesimga ega bo‘lgan plastinkada (Kirxgof va Timoshenko gipotezalari asosida) mavjud bo‘lgan o‘z to‘lqinlarining (eigenwaves) taqqoslamali o‘rganilishidir. Navye tenglamalari va fizik bog‘lanishlar asosida sektor shaklidagi to‘lqin yo‘lovchi uchun qisman hosilali olti differensial tenglamalar tizimi olinadi. Oddiy almashtirishlardan so‘ng, murakkab koeffitsiyentli differensial tenglamalar tizimi olinadi va bu to‘g‘ri chiziqlar usuli orqali yechiladi; bu esa murakkab arifmetikadagi Myuller usuli bilan ortogonal ko‘rib chiqish (sweep method) dasturiy vositasidan foydalanish imkonini beradi. Shuningdek, variatsion prinsip asosida murakkab koeffitsiyentli oddiy differensial tenglamalar tizimi keltirilgan. Sonli hisob-kitoblar asosida aniqlanishicha, birinchi modaning fazaviy tezligining haqiqiy qismlari Rayleigh to‘lqini tezligidan 20% gacha past bo‘ladi (ponasimon to‘lqin yo‘lovchidagi markaziy burchak segmentida) va keyin viskoelastik yarim tekislikda tarqaluvchi Rayleigh to‘lqiniga asimptotik tarzda yaqinlashadi. Kirxgof va Timoshenko plastinka nazariyasiga ko‘ra, xuddi shunday natijalar ponasimon plastinka uchun ham butun to‘lqin diapazonida kuzatildi. Elastiklikning dinamik nazariyasi va plastinka nazariyasiga (Kirxgof va Timoshenko gipotezalari asosida) oid natijalar orasidagi tafovut, ponaning uch burchagi 28° dan oshmagan holatda, 6% dan oshmaydi. Hisoblashlar shuni ko‘rsatdiki, kichik burchaklarda Kirxgof – Love va Timoshenko soddalashtirilgan nazariyalaridan butun to‘lqin diapazonida foydalanish mumkin.

Kalit so‘zlar: so‘nuvchi to‘lqin, viskoelastik silindr, sektor shaklidagi kesim, Navye tenglamasi, spektral chegaraviy masala, ortogonal tozalash.

АНАЛИЗ ВЯЗКОСТИ ОПРЕДЕЛЁННЫХ ВОЛН И ИХ РАСПРОСТРАНЕНИЯ

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Аннотация: Основная цель статьи — сравнительное исследование существования собственных волн в бесконечном вязкоупругом волноводе секторного сечения и в пластине переменного (клиновидного) сечения на основе гипотез Кирхгофа и Тимошенко, в зависимости от различных параметров объекта (волновое число и геометрические параметры). Используя уравнения Навье и физические соотношения, получена система из шести дифференциальных уравнений в частных производных для секторного волновода. После преобразований получена система дифференциальных уравнений с комплексными коэффициентами, решаемая методом прямых, что позволяет применять программный инструмент метода ортогонального прогонки в комбинации с методом Мюллера на комплексной арифметике. Также, на основе вариационного принципа, представлена система обыкновенных дифференциальных уравнений с комплексными коэффициентами. Численные расчёты показали, что действительные части фазовой скорости распространения первой моды на участке (центральный угол клиновидного волновода) до 20% ниже скорости волны Рэлея, и далее асимптотически приближаются к скорости Рэлея, распространяющейся в вязкоупругой полуплоскости. Аналогичные результаты получены и для клиновидной пластины по теории пластин Кирхгофа и Тимошенко на всём волновом диапазоне. Различия между результатами динамической теории упругости и теории пластин (по гипотезам Кирхгофа и Тимошенко) не превышают 6% при вершинах клина до 28°. Установлено, что при малых углах клина упрощённые теории Кирхгофа–Лавы и Тимошенко могут использоваться на всём волновом диапазоне.

Ключевые слова: затухающая волна, вязкоупругий цилиндр, секторное сечение, уравнение Навье, спектральная краевая задача, ортогональный метод прогонки.

1. Introduction

Elastic waves have been studied for more than a hundred years, but work in this direction does not stop, which indicates both the undying interest in this problem and the lack of knowledge of the subject. In work [1], the three-dimensional problem of wave propagation in an elastic layer. **A characteristic equation is obtained for the phase velocity of symmetric and antisymmetric oscillations. The limiting case is considered: the wavelength is very large and very small compared to the layer thickness.**

In the article [2] explored propagation of shear waves in a two-layer medium in an anti-plane setting. We consider the propagation of shear waves in a two-layer medium, when one layer is homogeneous and the other is inhomogeneous with exponential inhomogeneity. The dispersion equation has been studied. In monograph [3], the propagation of natural waves in extended waveguides is studied and dispersion relations are constructed. The change in phase and group wave velocities as a function of wave numbers is analyzed. In [4], at the limiting values of the parameters, the lower modes of the roots of the Pochhammer-Cree dispersion equation were obtained. As a result, they obtained a relationship between the phase velocity and the wave number. And in [5], these results were confirmed. They derive the dispersion equation from the system of differential equations of elasticity theory. The rod equation [6] was used to study the propagation of waves in a loaded reinforcement. In [7], with the help of a correction factor, dispersion curves are given for the phase and group velocities of a wave propagating in round rods. In the article [8], the Pochhammer-Cree dispersion equation was studied in a wide frequency range. The phase velocities of a one-dimensional wave were studied in [9] with and without dispersion. It was concluded in [9] that a one-dimensional problem (although with dispersion taken into account) will not characterize all dynamic processes occurring in a rod in a spatial formulation.

In the article [10] obtained differential equations of longitudinal free oscillations of a rod based on the variational principle of Hamilton. And also, in the article [11] the results of the above mentioned works were compared with the results obtained on the basis of the Pochhammer - Cree model. It is concluded that Bishop's model quite accurately describes the dispersion relation of a rod with a circular cross section, but the dispersion curve asymptotically tends to the shear wave velocity, while in the Pochhammer-Cree model, the asymptote is the Rayleigh wave velocity. In [12], the propagation of longitudinal waves in a rod in an axisymmetric formulation was studied and a characteristic equation was obtained.

In works [13, 14, 15] the solution of the Lamé equation was obtained using the Green's expansion, i.e. displacements are constructed through the potentials of longitudinal and transverse waves. It is emphasized that, according to the completeness theorem (Green's expansion), any wave process in an infinite elastic body can be described as a superposition of wave motions with longitudinal and transverse velocities waves. In [16], for the first time, for plates, a theory of Rayleigh wave propagation based on Kirchhoff's hypotheses was constructed, an analogue of the Rayleigh wave in the theory of bending vibrations of a plate. The plate was considered under conditions of a plane stress state. Since the problems of plane deformation of a cylindrical body and the plane stress state of a plate are mathematically identical. Then a planar wave of the Rayleigh type must exist in the plate, the speed of which is determined from the same equation as the speed of the Rayleigh wave, with the elastic Lamé constant μ replaced by the corresponding constant for the plate. This wave is called the Rayleigh-type bending wave or the Kononov wave [17]. In [18], with viscoelastic properties of materials explored infinitely long plates or strips of variable thickness.

Investigation of eigenwaves in a cylindrical viscoelastic waveguide with a sector cross section is an urgent task. Therefore, the present work is devoted to the study of the propagation of natural waves in a cylindrical viscoelastic waveguide with a sectoral cross section.

2. Methods

2.1 Statement of the problem and basic relationships

In this work, propagation of natural waves in an infinite viscoelastic cylinder with a sector cross section (Fig. 1) is considered. The waveguide has a collinear axis directed along the Oz axis. The problem of analyzing the spectra of normal elastic waves along the considered waveguide is formulated using the relations of a spatial linear mathematical model of the dynamic stress-strain state of deformable bodies, taking into account viscoelastic properties, in a cylindrical coordinate system.

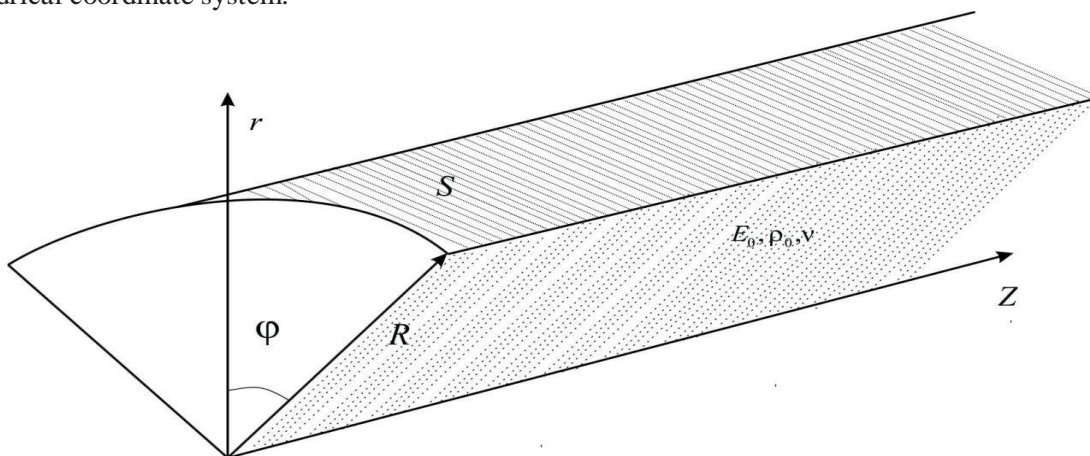


Fig.1. Calculation scheme of an infinite viscoelastic cylinder with sectoral cross section.

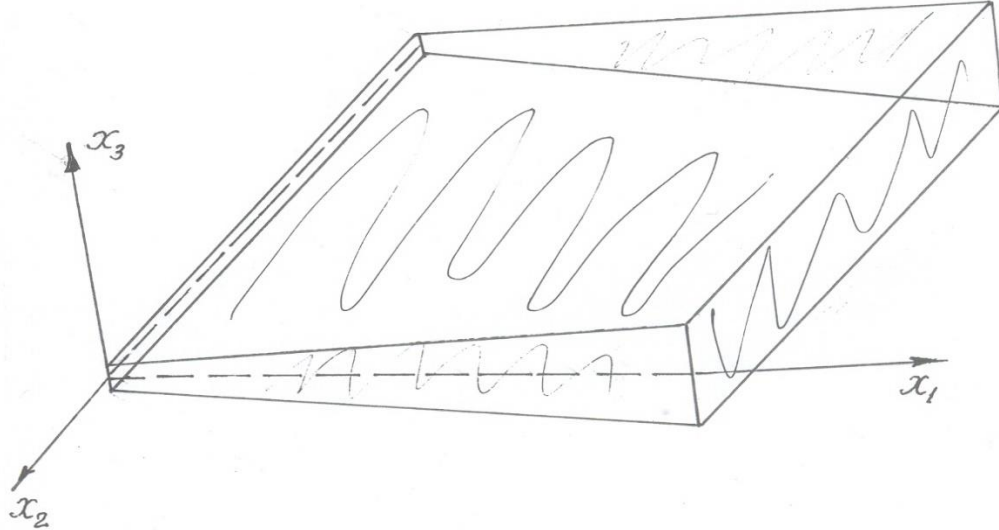


Fig.2. Calculation scheme for a plate of variable thickness.

These relations are formulated for the projections of the dimensionless vector of dynamic elastic wave displacements on the axes of the cylindrical coordinate system $\{u_r, u_\varphi, u_z\}$, as well as for the dimensionless characteristics of the stress-strain state of the object under consideration on the main areas of the cylindrical coordinate system $\{\sigma_{rr}, \sigma_{r\varphi}, \sigma_{\varphi\varphi}, \sigma_{zz}, \sigma_{rz}, \sigma_{\varphi z}\}$. The basic equations of motion of an elastic medium occupying region B are given by three groups of relations[nineteen]:

$$\frac{\partial \sigma_{ik}}{\partial x_k} + \rho g_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$\varepsilon_{ik} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right), \quad \sigma_{ik} = \tilde{\lambda} \theta \delta_{ik} + 2\tilde{\mu} \varepsilon_{ik}.$$

Here σ_{ik} -elements of the stress tensor, ε_{ik} - strain tensor elements, θ - volumetric deformation, $\tilde{\lambda}$ and $\tilde{\mu}$ - complex quantities:

$$\tilde{\lambda}[f(t)] = \frac{\nu E_0}{(1+\nu)(1-2\nu)} \left[f(t) - \int_0^t R_\lambda(t-\tau) f(\tau) d\tau \right]; \quad (2)$$

$$\tilde{\mu}[f(t)] = \frac{\nu E_0}{2(1+\nu)} \left[f(t) - \int_0^t R_\mu(t-\tau) f(\tau) d\tau \right]$$

where ν - Poisson's ratio is taken constant; $\varphi(t)$ is an arbitrary function of time; $R_\lambda(t-\tau), R_\mu(t-\tau)$ - relaxation nuclei; E_0 is the instantaneous modulus of elasticity.

We take the integral terms in (2) small. Then the function $f(t) = \psi(t) e^{-i\omega_R t}$, where $\psi(t)$ - slowly changing function of time, ω_R is a real constant. Next, applying the freezing procedure [16], we replace relations (2) with approximate ones of the form

$$\bar{E}f = E \left[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R) \right] f, \quad (3)$$

where $\Gamma^C(\omega_R) = \int_0^\infty R(\tau) \cos \omega_R \tau d\tau$, $\Gamma^S(\omega_R) = \int_0^\infty R(\tau) \sin \omega_R \tau d\tau$ are, respectively, the cosine and sine

Fourier images of the material relaxation kernel. As an example of a viscoelastic material, we take a three-parametric relaxation kernel: $R(t) = A e^{-\beta t} / t^{1-\alpha}$.

Relations (1), (2), (3) after identical algebraic transformations are reduced to a system of six differential equations resolved with respect to the first derivative with respect to the radial coordinate

$$\begin{cases}
\frac{\partial u_r}{\partial r} = \frac{1}{K} \sigma_{rr} - \frac{\lambda}{K} \left(\frac{1}{r} \frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} \right); \\
\frac{\partial u_\varphi}{\partial r} = \frac{1}{\mu} \sigma_{r\varphi} - \frac{1}{r} \left(\frac{\partial u_r}{\partial \varphi} - u_\varphi \right); \\
\frac{\partial u_z}{\partial r} = \frac{1}{\mu} \sigma_{rz} - \frac{\partial u_r}{\partial z}; \\
\frac{\partial \sigma_{rr}}{\partial r} = \rho \frac{\partial^2 u_r}{\partial t^2} - \frac{\tilde{A}}{r} - \frac{1}{r} \frac{\partial \sigma_{r\varphi}}{\partial \varphi} - \frac{\partial \sigma_{rz}}{\partial z}; \\
\frac{\partial \sigma_{r\varphi}}{\partial r} = \rho \frac{\partial^2 u_\varphi}{\partial t^2} - \frac{1}{r} \frac{\partial}{\partial \varphi} [\sigma_{rr} - \tilde{A}] - \frac{2\sigma_{r\varphi}}{r} - \frac{\partial}{\partial z} \tilde{B}; \\
\frac{\partial \sigma_{rz}}{\partial r} = \rho \frac{\partial^2 u_z}{\partial t^2} - \frac{\partial}{\partial z} \left[\sigma_{rr} - 2\mu \left(\frac{\partial u_r}{\partial r} - \frac{\partial u_z}{\partial z} \right) \right] - \frac{\sigma_{rz}}{r} - \frac{1}{r} \frac{\partial}{\partial \varphi} \tilde{B};
\end{cases} \quad (4)$$

where the notation

$$\tilde{A} = 2\mu \left[\frac{\partial u_r}{\partial r} - \frac{1}{r} \left(\frac{\partial u_\varphi}{\partial \varphi} + u_r \right) \right]; \quad \tilde{B} = \mu \left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \varphi} \right).$$

Boundary conditions for φ , for an arbitrary angle of the sector in the case of a free side surface, should be written as:

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \quad \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0, \quad (5)$$

the boundary conditions along the radius are set in the form:

$$r = r_0 \rightarrow 0, R: \quad \sigma_{rz} = \sigma_{rr} = \sigma_{r\varphi} = 0. \quad (6)$$

Here φ_0 is the angle at the top of the wedge. Own wave, propagate along the longitudinal axis z . The main task (1), (4), (5) and (6) is to solve the equation satisfying periodicity in z and in time.

The periodicity conditions make it possible to eliminate the dependence of the main unknowns on time and the axial coordinate z using the following substitution variables:

$$\begin{aligned}
\sigma_{rr} &= \sigma(r, \varphi) e^{i(\kappa z - \omega t)}; u_r = w(r, \varphi) e^{i(\kappa z - \omega t)}; \\
u_\varphi &= v(r, \varphi) e^{i(\kappa z - \omega t)}; u_z = u(r, \varphi) e^{i(\kappa z - \omega t)}, \\
\sigma_{rz} &= \tau_z(r, \varphi) e^{i(\kappa z - \omega t)}, \sigma_{r\varphi} = \tau_\varphi(r, \varphi) e^{i(\kappa z - \omega t)}.
\end{aligned} \quad (7)$$

Here $\sigma(r, \varphi)$, $\tau_\varphi(r, \varphi)$, $\tau_z(r, \varphi)$, $w(r, \varphi)$, $v(r, \varphi)$, $u(r, \varphi)$ - complex value, which means amplitude functions, ω - complex circular frequency, it is required to determine, ($\omega = 2\pi\nu$, ν is the complex frequency of oscillations (complex value); $\lambda = 2\pi/\alpha$ - wavelength, s - phase velocity of waves (complex value), $\omega = \alpha c$, α - wave number.

Taking into account (7), the system of equations (4) takes the form:

$$\left\{ \begin{array}{l} w' = \frac{\sigma}{K} - \frac{\lambda}{K} \left(ku + \frac{1}{r} \left(w + \frac{\partial v}{\partial \varphi} \right) \right) \\ v' = \frac{\tau_\phi}{\mu} + \frac{1}{r} \left(v - \frac{\partial w}{\partial \varphi} \right) \\ u' = \frac{\tau_z}{\mu} + kw \\ \sigma' = -\omega^2 \rho w + \frac{1}{r} \left(A - \frac{\partial \tau_\phi}{\partial \varphi} \right) - k\tau_z \\ \tau'_\phi = -\omega^2 \rho v - \frac{1}{r} \left(\frac{\partial(A + \sigma)}{\partial \varphi} + 2\tau_\phi \right) - kB \\ \tau'_z = -\omega^2 \rho u - \frac{1}{r} \left(\frac{\partial B}{\partial \varphi} + \tau_z \right) + k(\sigma + 2\bar{\mu}(ku - w')) \end{array} \right. \quad (8)$$

where $A = 2\bar{\mu} \left(\frac{1}{2} \left(\frac{\partial v}{\partial \varphi} + w \right) - w' \right)$, $B = \bar{\mu} \left(\frac{1}{r} \frac{\partial u}{\partial \varphi} - kv \right)$.

Boundary conditions (6) are transformed similarly:

$$r = \mathbf{0}, R: \sigma = \tau_\phi = \tau_z = \mathbf{0}. \quad (9)$$

It is easy to see that the components of the stress tensor $\sigma_{\varphi\varphi}$, $\sigma_{\varphi z}$ and σ_{zz} are expressed in terms of the main unknowns by the formulas:

$$\sigma_{\varphi\varphi} = \sigma_{rr} + 2\bar{\mu} \left(\frac{1}{r} \frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r}{r} - \frac{\partial u_r}{\partial r} \right), \sigma_{\varphi z} = \bar{\mu} \left(\frac{\partial u_z}{\partial \varphi} + \frac{\partial u_\varphi}{\partial z} \right) \quad (10)$$

$$\sigma_{zz} = \sigma_{rr} + 2\bar{\mu} \left(\frac{\partial u_z}{\partial z} - \frac{\partial u_r}{\partial r} \right).$$

Then, taking into account the first equation of system (5), the boundary conditions (9) take the form:

$$\sigma_\varphi = A + \sigma_r = a\sigma_r + b \frac{1}{r} \left(\frac{\partial v}{\partial \varphi} + w \right) + cku = 0$$

$$\varphi = -\frac{\varphi_0}{2}, \frac{\varphi_0}{2}; \tau_\phi = \mathbf{0} \quad (11)$$

$$B = \bar{\mu} \left(\frac{\partial u}{r \partial \varphi} - kw \right) = 0,$$

where

$$a = 1 + \frac{2\bar{\mu}}{k} \quad b = 2\bar{\mu} \left(1 + \frac{\lambda}{K} \right) \quad c = 2\mu \frac{\lambda}{k}.$$

The boundary value problem for a system of partial differential equations (7), (8), (10) is reduced to a problem for a system of complex ordinary differential equations using the method of lines, which allows using the orthogonal sweep method software.

The method of lines belongs to the semi-discrete methods. Semi-discrete methods are based on the following idea: partial discretization of a partial differential equation is not complete, but not in all independent variables. For example, you can leave r as a continuous variable and discretize only the variable. The essence of the straight line method is that the rectangular area of the function of the main unknowns is covered with straight lines parallel to the r axis and equidistant from each other. Numerical values in given fields are sought only along these lines, and the derivative in the direction φ is replaced by approximate finite differences.

Note that the choice of boundary conditions on slotted surfaces of the form (9) was determined primarily by the possibility of separating variables over the coordinates r and φ , which greatly simplifies the solution of the original problem. According to the method of lines, the rectangular area of the function of principal unknowns is covered with lines parallel to the r -axis and equidistant from each other. The solution is sought only along these lines, and the derivative with respect to φ is replaced by a finite difference approximation. Approximate formulas of the second order

for the first and second derivatives have the form

$$y_{i,\varphi} \cong \frac{y_{i+1} - y_{i-1}}{2\Delta} \cong \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta} \cong \frac{3y_i - 4y_{i-1} + y_{i-2}}{2\Delta}$$

$$y''_{i,\varphi} \cong \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta^2}, \quad (12)$$

where i changes from 0 to $N+1$ ($i=0, \overline{N+1}$), -projection of an unknown function onto a straight line with number i ; Δ is the partitioning step along the φ coordinate.

As a result of discretization, the vector of main unknowns of the total dimension $6N$ can be written as:

$$Y = (\{w_i\}, \{v_i\}, \{u_i\}, \{\sigma_{ri}\}, \{\tau_{\varphi i}\}, \{\tau_{zi}\})^T \cdot (i = \overline{1, N}) \quad (13)$$

Central differences (12) are used for internal lines ($1 < i < N$), the left and right differences allow taking into account the boundary conditions in φ . In the first case, the derivative with respect to φ in the right parts of the system of equations (8) is expressed by the formulas:

$1 < i < N$:

$$w_{i,\varphi} = (w_{i+1} - w_{i-1})/2\Delta, \quad u_{i,\varphi} = (u_{i+1} - u_{i-1})/2\Delta, \quad (14)$$

$$v_{i,\varphi} = (v_{i+1} - v_{i-1})/2\Delta, \quad \tau_{\varphi i,\varphi} = (\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)})/2\Delta,$$

$$\tau_{\varphi i,\varphi} = (\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)})/2\Delta$$

$$\sigma_{\varphi i,\varphi} = a(\sigma_{i+1} - \sigma_{i-1})/2\Delta + \frac{b}{r}[(v_{i+1} - 2v_i + v_{i-1})/\Delta^2 + w_{i,\varphi}] + ck u_{i,\varphi}$$

$$B_i = (u_{i+1} - 2u_i + u_{i-1})/\Delta^2 / k - kv_{i,\varphi}.$$

Boundary conditions at $\varphi = -\frac{\varphi_0}{2}$ is taken into account in the above equations corresponding to the straight lines $i = I$. For the main unknowns, not included in the boundary conditions, w_i, v_i, u_i , the right differences of the first relation (11) are used:

$$w_{i,\varphi} = (-3w_1 + 4w_2 - w_3)/2\Delta, \quad v_{i,\varphi} = (-3v_1 + 4v_2 - v_3)/2\Delta,$$

$$u_{i,\varphi} = (-3u_1 + 4u_2 - u_3)/2\Delta. \quad (15)$$

For variable tangential stress τ_{φ} , conditions (10) are used using the central difference

$$\tau_{\varphi i,\varphi} \cong (\tau_{\varphi_2} - \tau_{\varphi_0})/2\Delta = -\tau_{\varphi_2}/2\Delta. \quad (16)$$

When approximating the differential of the function B with respect to φ , then the first and third of the conditions (10) σ_{φ} takes the form

$$\sigma_{\varphi i,\varphi} \cong (\sigma_{\varphi_2} - \sigma_{\varphi_0})/2\Delta = \sigma_{\varphi_2}/2\Delta = \left(a\sigma_{\varphi_2} + \frac{b}{r}[(v_3 - v_1)/2\Delta + w_2] - ck u_2 \right) / 2\Delta$$

$$B_{i,\varphi} \cong (B_2 - B_0)/2\Delta = B_2/2\Delta = [(u_3 - u_1)/2\Delta / r - kv_2] / 2\Delta \quad (17)$$

It is also possible to represent the derivatives for the boundary conditions at $\varphi = \frac{\varphi_0}{2}$, and will represent a straight line with number $i = N$. The only difference is the replacement of right finite differences by left ones:

$i = N$:

$$w_{i,\varphi} = (3w_N - 4w_{N-1} + w_{N-2})/2\Delta$$

$$v_{i,\varphi} = (3v_N - \dots)/2\Delta$$

$$U_{i,\varphi} = (U_{i+1} - U_{i-1})/2\Delta$$

$$u_{i,\varphi} = (3u_N - \dots)/2\Delta \quad (18)$$

$$\begin{aligned}\tau_{\varphi,\varphi} &= -\tau_{\varphi(N-1)}/2\Delta \\ \tau_{\varphi_i,\varphi} &= (\tau_{\varphi(i+1)} - \tau_{\varphi(i-1)})/2\Delta \\ \sigma_{i,\varphi} &= \left(a\sigma_{N-1} + \frac{b}{r}[(v_N - v_{N-2})/2\Delta + w_{N-1}] + ck u_{N-1} \right) / 2\Delta = -\frac{\sigma_{N-1}}{2\Delta} \\ B_{i,\varphi} &= -[(u_N - u_{N-2})/2\Delta/r - kv_{N-1}] / 2\Delta = -\frac{B_{N-1}}{2\Delta}\end{aligned}$$

If we take into account the conditions of antisymmetry of vibrations (transverse) of the plate at $\varphi = 0$, then halve the number of lines in the equations:

$$w = u = \sigma_\varphi = 0. \quad (19)$$

The corresponding difference relations, taking into account conditions (19), can be written as:

$$\begin{aligned}i=N: w_{i,\varphi} &= -w_{N-1}/2\Delta \quad u_{i,\varphi} = -u_{N-1}/2\Delta \\ v_{i,\varphi} &= (3v_N - \dots)/2\Delta \quad (20) \\ \tau_{\varphi_i,\varphi} &= (3\tau_{\varphi N} - 4\tau_{\varphi(N-1)} + \tau_{\varphi(N-2)})/2\Delta \\ \sigma_{i,\varphi} &= -\left(a\sigma_{N-1} + \frac{b}{r}[(v_N - v_{N-2})/2\Delta + w_{N-1}] + ck u_{N-1} \right) / 2\Delta = -\frac{\sigma_{N-1}}{2\Delta} \\ B_{i,\varphi} &= -(-2u_N + u_{N-1})/\Delta^2/r - kv_{i,\varphi}.\end{aligned}$$

The system of ordinary differential equations with complex coefficients, according to (19), has the form:

$$\begin{aligned}w'_i &= \sigma_i/k - a(ku_i + (w_i + v_{i,\varphi})/R); \\ v'_i &= \tau_{\varphi i} + (v_i - w_{i,\varphi})/R \quad (21) \\ u'_i &= \tau_{zi} + kw_i \\ \sigma'_i &= -\omega^2 w_i + [2((w_i + v_{i,\varphi})/R - w'_i) - \tau_{\varphi_i,\varphi}]/r - k\tau_{zi} \\ \tau'_{zi} &= -\omega^2 u_i - (B_{i,\varphi} + \tau_{zi})/r + k(\sigma + 2(ku_i - w'_i)); \\ \tau'_{\varphi i} &= -\omega^2 v_i + (\sigma_{i,\varphi} + 2\tau_{\varphi i})/r - k(u_{i,\varphi}/R - kv_i).\end{aligned}$$

In differential equations (21) with complex coefficients, the expressions for the derivatives w'_i , v'_i , u'_i , σ'_i , τ'_{zi} , $\tau'_{\varphi i}$ are obtained from relations (19) - (21) depending on the boundary conditions along the coordinate φ . Also, conditions equivalent to conditions (19) on the free surface are posed, and forming together with equations (21) the boundary value problem, are obtained in the form

$$B_i = 0, \quad \tau_{\varphi i} = 0, \quad \sigma_\varphi = 0, \quad (i=1,N). \quad (22)$$

Thus, the original spectral problem (21), (22) with the help of discretization in the coordinate φ , according to the method of lines, is reduced to a canonical problem, for the solution of which we apply the previously used orthogonal sweep method.

The system of ordinary differential equations of the first order with complex coefficients, for a wedge-shaped plate, resolved with respect to derivatives, has the form[15]:

$$\begin{cases} z'_1 = z_2; \\ z'_2 = -\frac{6(1-\nu)}{h^3} z_3 + \nu\kappa^2 z_1; \\ z'_3 = z_4 - \frac{h^3 \Gamma_k}{3} \kappa^2 z_2; \\ z'_4 = \nu\kappa^2 z_3 + \frac{(1+\nu)h}{6} \kappa^4 z_1 - h\left(\frac{\omega}{C_s}\right)^2 \Gamma_k z_1; \end{cases} \quad (23)$$

The boundary conditions for this system can be written as follows:

a) the free left edge of the plate:

$$z_3(0) = z_4(0) = 0 \quad (24)$$

b) free right edge of the plate:

$$z_3(l) = z_4(l) = 0 \quad (25)$$

c) pinched right edge of the plate:

$$z_1(l) = z_2(l) = 0 \quad (26)$$

Thus, the spectral boundary value problem (23-26) has been formed with respect to the parameter ω describing the propagation of bending flat edges-face waves in a Kirchhoff plate.

When the Timoshenko hypothesis is fulfilled, then the system of partial differential equations (23) takes the following form:

$$\left\{ \begin{array}{l} z_1' = z_2 + \frac{z_n}{\chi h}; \\ z_2' = -\nu \kappa z_3 - \frac{6(1-\nu)}{3} z_5; \\ z_3' = \kappa z_2 - \frac{12}{h^3} z_6; \\ z_4' = \chi h \kappa z_3 + \kappa^2 \left(\chi h - \frac{hc^2}{\Gamma_n} \right) z_1; \\ z_5' = -\kappa z_6 + z_4 + \frac{h^3}{12\Gamma_n} \omega^2 z_2; \\ z_6' = -\chi h \kappa z_1 - \left[\chi h + \frac{\kappa^2 h^3}{12\Gamma_n} \left(2(1+\nu) - \frac{c^2}{\Gamma_n} \right) \right] z_3 + \nu \kappa z_5. \end{array} \right. \quad (27)$$

The boundary conditions for this system can be written as follows:

a) the free left edge of the plate:

$$z_4 = z_5 = z_6 = 0, \quad x = 0; \quad (28)$$

b) free right edge of the plate:

$$z_4 = z_5 = z_6 = 0, \quad x = l; \quad (29)$$

c) pinched right edge of the plate:

$$z_1 = z_2 = z_3 = 0, \quad x = l; \quad (30)$$

Thus, the spectral boundary value problem (27-30) has been formulated with respect to the parameter ω describing the propagation of bending plane edge waves in the Timoshenko plate.

3. Results and analysis

As an example of a viscoelastic material, we take a three-parametric relaxation kernel: $R_\lambda(t) = R_\mu(t) = A e^{-\beta t} / t^{1-\alpha}$. Dimensionless quantities are chosen in such a way that the shear rate C_s , density ρ and outer radius R have single values, Poisson's ratio $\nu = 0.25$, and core parameters $A = 0.048$; $\beta = 0.05$; $\alpha = 0.1$

The calculations were carried out in dimensionless parameters. The change in the complex phase velocity $C_0 = C_{0R} + iC_{0I}$ from corners $m^{\square} = m\varphi$ at different values of the instantaneous modulus of elasticity. The calculation results are shown in Fig.3 and Fig.4.

Figure 3 shows the change in the real part of the complex phase velocity C_0 from m^{\square} at different values of the instantaneous modulus of elasticity: ($C_{0R} = C_{0R}^{\square} / C_R$, C_R - speed of Rayleigh's ox) 1. $E=0.01$; 2. $E=0.05$; 3. $E=0.1$; 4. $E=0.8$, where $E = E_0 / E_p$, E_p - modulus of elasticity of steel). From Fig. 3 it can be seen that with an increase m^{\square} the phase velocities increase and asymptotically approach the Rayleigh wave. Figure 4 shows the change in the imaginary part of the complex phase velocity from m^{\square} at different values of the instantaneous modulus of elasticity

$$(C_{0R} = C_{0R}^{\square} / C_R, C_R - \text{speed of Rayleigh's ox}): 1. E=0.01; 2. E=0.05; 3. E=0.1; 4. E=0.8, \text{ where } E = E_0 / E_p,$$

E_p - modulus of elasticity of steel).

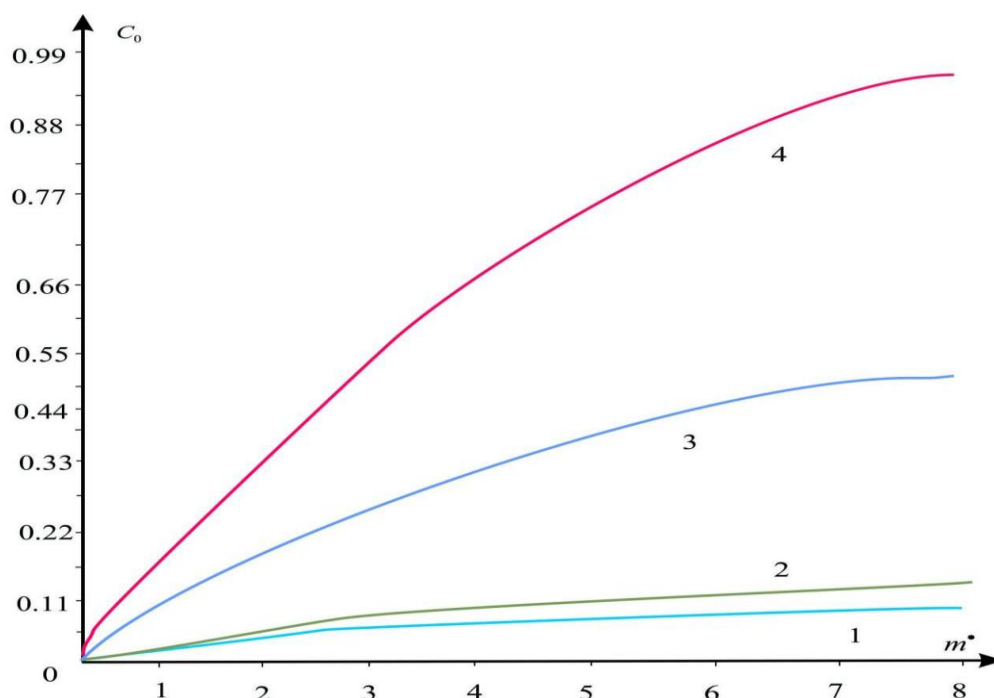


Fig. 3. Dependence of the real part of the complex phase velocity on at different values of the instantaneous modulus of elasticity.

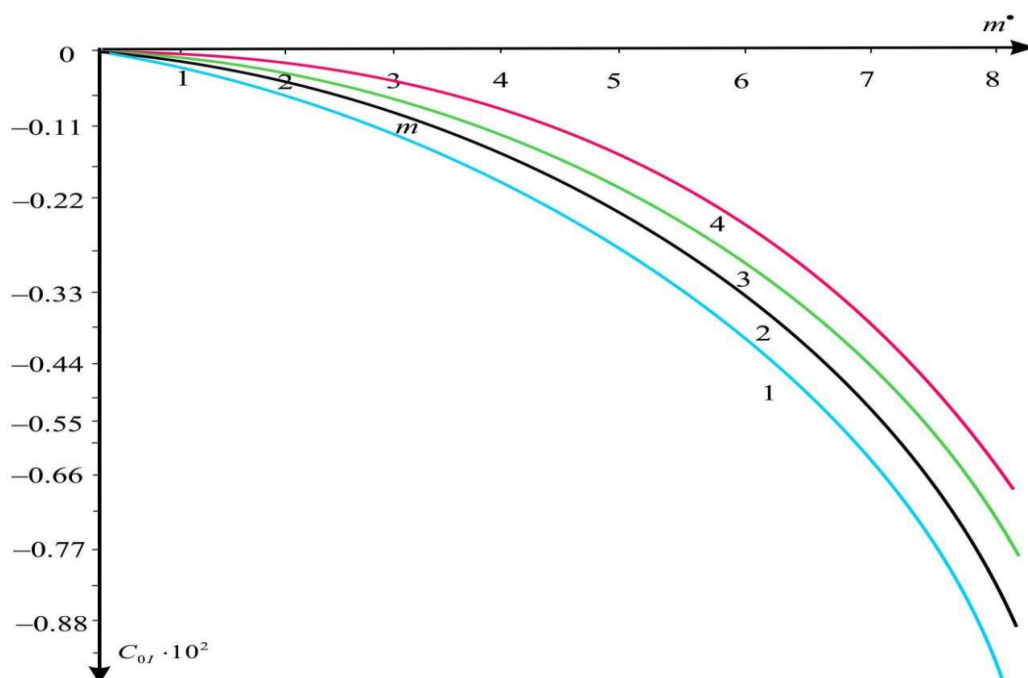


Fig. 4. Dependence of the imaginary part of the complex phase velocity on at different values of the instantaneous modulus of elasticity.

For a Kirchhoff plate of variable thickness, we studied the first five modes with the lowest complex phase propagation velocities $C = C_R + iC_I$, where C_R - phase velocities of propagating waves; C_I - damping speed. Figure 5 shows the dispersion curves of the first mode depending on the thickness, which varies linearly. It is assumed here that both edges of the plate are free. Straight line I corresponds to a constant thickness $h_1=h_2=0.1$. In this case, the plate oscillates like a rod. Curve II - variant $h_1=h_2/2=0.05$; curve III - variant $h_1=h_2/100=0.001$, curve IV-

$h_1 = h_2 / 1000 = 0,0001$ and $E_{\min} = 6,9 \cdot 10^6 \text{ } \kappa / M^2$, $E_{\max} = 6,9 \cdot 10^8 \text{ } \kappa / M^2$, $\beta = 10^{-4}$. It is seen that at $\kappa > 9$ damping speed increases with k . For a plate of constant thickness C_r for a segment $10^{-4} < C < 70$ decreases in a straight line. It can be seen that the dependence of the damping coefficient on the wave number starts from the wave number 3.6. With the entrainment of the wave number, the damping coefficient tends to the side of decrease.

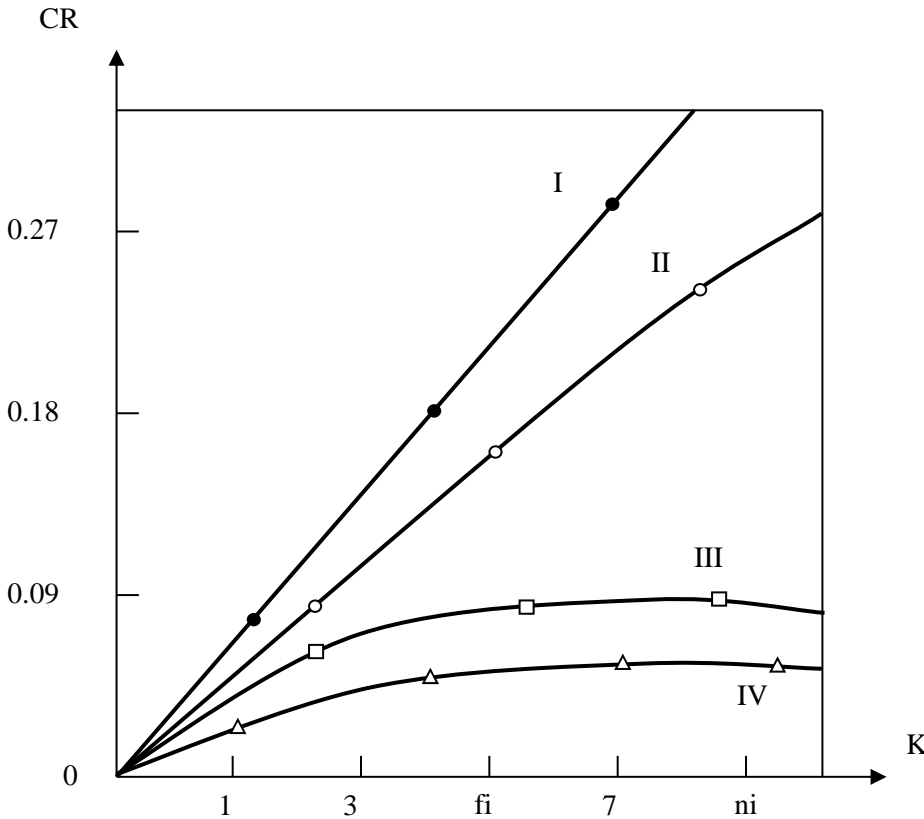


Fig.5. Dispersion curves of the first mode

I. $h_1=h_2=0.1$; II. $h_1=h_2/2=0.05$; III. $h_2/100 = 0.001$; IV. $h_1=h_2/1000=0.001$

The calculation results obtained by equations (8)-(11) are compared with approximate equations obtained on the basis of the Kirchhoff and Timoshenko hypotheses (23)-(30). Based on the numerical results, the following relationship was obtained

$$C_0 = C_R \sin(m\varphi), \quad (31)$$

at $m\varphi < 90^\circ$ ($m = 1, 2, \dots$), which is given in [16, 18]. The waves that appeared in the acute angle of the wedge were called wedge waves or Troyanovsky-Safarov waves. The damping decrements of these waves turned out to be small numbers, but they change rapidly, i.e. turned out to be nonmonotonic functions of the parameters of harmonic waves.

When the viscosity C_0 is taken into account, the velocities of these waves decrease to 15%. As the opening angle decreases, the corresponding damping coefficients also decrease exponentially. For the damping factor, based on the numerical results, the following empirical relationship is proposed

$$C_{OI} = -C_R e^{-3.2m} \sin(m\varphi). \quad (32)$$

In table. Figure 1 shows the limiting values of the real part of the phase velocity of the first edge mode as a function of the wedge angle. Columns 5-6 show the phase velocities for various boundary conditions found within the framework of the above-described method for calculating a three-dimensional wedge. Column 5 corresponds to the calculation option with three internal straight lines ($N = 3$) and boundary conditions (11), column 6 corresponds to the boundary conditions:

$$\varphi = -\frac{\theta_0}{2} : \sigma_{\varphi\varphi} = \sigma_{\varphi r} = \sigma_{\varphi z} = 0; \quad (32)$$

$$\varphi = 0 : u_r = u_z = \sigma_{\varphi\varphi} = 0.$$

Column 6 shows the results of calculations obtained by formula (31).

Also in columns 3 and 4, according to the theory of Kirchhoff and Timoshenko plates, the limiting values of the real part of the phase velocity of the first edge mode, obtained in [20, 21], are given.

According to the numerical results and the results presented in Table 1, we can conclude that the calculation options according to the Kirchhoff, Timoshenko methods and the three-dimensional theory are consistent within 7% at the corners of the wedge. base thickness does not exceed 0.5 (wedge angle $\varphi = 28^\circ$). Note that the limiting phase velocity for the angle $\varphi = 90^\circ$ was calculated in [15], where the value 0.901 ($\nu = 0.25$). Thus, in wedge-shaped waveguides with a sufficiently small wedge angle, in contrast to rectangular waveguides, it is permissible to use the theory of Kirchhoff-Love plates in the analysis of the dispersion dependence of the first mode.

Table 1. Limiting values of the phase velocity of the first edge mode depending on the wedge angle

R	θ_0	By Krichhoff hypothesis	By Timoshenko's hypothesis	By three-dimensional theory, boundary conditions (11)	By 3D theory, boundary conditions	By formula (3.36) $R_\lambda = R_\mu = 0$
0.2	one hundred	0.2260	0.1862	0.1761	0.2542	0.1827
0.3	150	0.3802	0.2964	0.2982	0.3082	0.2763
0.5	280	0.5385	0.4329	0.4626	0.4755	0.4335
0.7	400	0.7014	0.5535	0.5923	0.6059	0.5746
1	600	1.0144	0.6718	0.7294	0.7414	0.7368
2	900	2.8637	0.8441	0.8792	0.8949	0.9212

It is known that at sufficiently small wedge angles, the theory of Kirchhoff plates can be used. When analyzing the dispersion dependence of the first mode, as well as the modes of oscillations near an acute wedge angle, they are also satisfactorily described by the theory of plates. [15]. This phenomenon should be considered as a characteristic of the dynamic behavior of a waveguide of variable thickness. In the case of a sector cylinder, the first mode has a limiting frequency, and the phase velocity tends to infinity. At large wave numbers, the limiting phase velocity of this mode also corresponds to the velocity of the Rayleigh wave. When locking, the axial displacements are equal to zero, and the oscillations of an infinite viscoelastic cylinder of a sectorial cross section occur in a plane-strained state. In the second mode, only the real and imaginary parts of the axial displacement are observed at the cut-off frequency, circumferential and radial displacements are equal to zero. In contrast to edge waves on a sharp wedge, viscoelastic waves on an infinite cylinder with sectoral cross section do not have a limiting solution, since the outer radius tends to infinity.

4. Conclusions

Based on the results of the research, the following conclusions were made:

1. On the basis of numerical results, it has been established that the propagation velocities in the wedge-shaped plates, subject to the Krichhoff-Love and Timoshenko hypotheses, differ by up to 6%. To fulfill these hypotheses, the wedge apex angle must be up to 28° . If the vertex angle is in $28^\circ < \varphi < 90^\circ$ Within the limits, the results differ by up to 20%.

2. Asymptotic dependences describing the phase velocities of wedge-shaped waves (31) and (32), the so-called Troyanovsky-Safarov formulas, are obtained.

3. The damping decrements turned out to be small numbers, but they change rapidly, i.e. turned out to be nonmonotonic functions of the waveguide parameters. When viscosity is taken into account, COR is reduced to

15%. As the opening angle decreases, the corresponding damping coefficients also decrease exponentially.

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